

Chapter 5

Time value of money

أهم ما يتر
يكون المساج
ولقد اتم كمان

Present value

(القيمة الحالية)

future value

(القيمة المستقبلية)

النقدية الحالية $\$10,000$ التي يرجى اقتضاها بعد 5 سنوات

قيمة اليوم

قيمة المال التي يتغير مع مرور الزمن

لأنه إن أمكنه لقيامه بالإنفاق اقتضاها اليوم

* المصاريف التي يرجى اقتضاها بعد 5 سنوات \rightarrow حشرت الإيراد

التي يمكن أن يحصل على مئودون المصاريف

discounting \rightarrow present value \rightarrow بالأسفل

\Rightarrow We calculate the present value of a certain amount of money that is expected to be received at same future time.

12) Future value: we calculate the future value for a certain amount of money that we have today (Compounding)

فائدة مركبة (مستوى بلات) (simple interest) الفائدة بضيفها على آخر مبلغ (مستوى ال simple)

Simple interest

$PV = \$1000$
 $r = 5\%$
 $n = 2 \text{ years}$
 $FV = ??$

$\Rightarrow r = 1000 \times 5\% \times 2 = \100
 $FV = 1000 + 100 = \$1100$

Compounded interest:

$PV = \$1000$
 $r = 5\%$
 $n = 2 \text{ years}$
 $FV = ??$

$\Rightarrow r = 5\% \times 1000 = 50$
 after 1 year = $1000 + 50 = 1050$
 after year 2: $r = 5\% \times 1050 = \$52.5$

من 1000 (simple interest) فائدة مركبة بضيفها على آخر مبلغ (مستوى ال simple) فائدة مركبة بضيفها على آخر مبلغ (مستوى ال simple)

$\Rightarrow FV = 1000 + 52.5 = \boxed{\$1102.5}$

$(1050 + 52.5)$

* Basic cash flow patterns

- ① Single amount
- ② Annuity = equal cash flows (could be in/out) - equal amount of money.

- ③ Mixed stream = unequal cash flows

□ Calculating future value of a single amount

↓
Compounding of interest

قلم

$$FV = PV (1+r)^n$$

future value present value rate of return # of periods

$$S = P(1+r)^n$$

e.g (Page 215)

$$PV = \$800$$

$r = 6\%$ compounded annually.

$$n = 5 \text{ years}$$

$$FV = ??$$

The higher

(n, r)

The higher FV

$$FV = PV (1+r)^n$$

$$= 800 (1 + 0.06)^5$$

$$= ~~\$1,070.58~~ \quad \boxed{\$1,070.58}$$

2 Calculating the present value of single amount

$$FV = PV (1+r)^n \Rightarrow PV = \frac{FV}{(1+r)^n}$$

⇒ The higher the (n, r) the longer the lower, the present value الوقت على كسبه

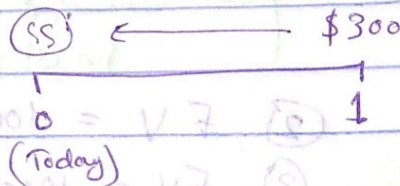
eg Page 217: $FV = \$300$ $n = 1$ year

$r = 6\%$

- discount rate : استوفاه من زمان
- interest rate - opportunity cost.
- rate of return.

$PV = ??$

$$PV = \frac{FV}{(1+r)^n}$$



$$PV = \frac{300}{(1.06)^1} = \$283.02$$

3 Calculating future of an annuity:

Annuity (equal cash flows)

ordinary annuity. annuity due.

(فجوة) في

(فجوة) في

assumes that cash inflows/outflows happen at the end of each period.

assumes that cash inflows/outflows happen at the beginning of each period

* Calculating future of an ordinary annuity (at the end)

eg (page 221)

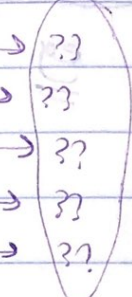
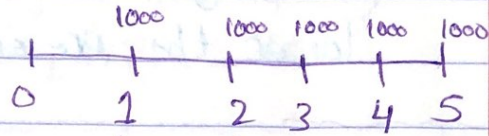
CF = \$1000 / year

ordinary annuity

n = 5 years

r = 7%

Future Value of Annuity ← FVA = ??



بقرینہ آج سے ہر سال 1000 کے لیے 5 سالوں کے لیے فوٹو لینا

$$① FV = PV (1+r)^n = 1000 (1+0.07)^5 = \$1,310.8$$

$$② FV = 1000 (1+0.07)^3 = \$1,225.04$$

$$③ FV = 1000 (1+0.07)^2 = \$1,144.9$$

$$④ FV = 1000 (1+0.07)^1 = \$1070$$

$$⑤ FV = 1000 (1+0.07)^0 = \$1000$$

1000, 1070, 1144.9, 1225.04, 1310.8

$$\Rightarrow FVA = 1,310.8 + 1,225.04 + 1,144.9$$

$$+ 1070 + 1000 = \$5750.74$$

Dec 10, 2019

Tuesday

* $FVA = CF \left(\frac{(1+r)^n - 1}{r} \right)$

\downarrow future value of an ordinary annuity \downarrow cash flow (in/out) \downarrow number of periods
 r \downarrow required of return interest rate

eg page 221 : CF = \$1000 / year

n = 5 years

r = 7%

كلمة مثال الحاضرة، لاضح
 بس مختصرة بهي لاضح
 (FVA)

$$FVA = CF \left(\frac{(1+r)^n - 1}{r} \right) = 1000 \left(\frac{(1+7\%)^5 - 1}{7\%} \right)$$

= \$5,750.74

لاحظ أخي: نفس الجواب (٤)

[4] Calculating present value of an ordinary annuity :

* Example page 223: at the end of the year \Rightarrow ordinary
 ← لأنها في نهاية الفترة يكون

CF = \$700 / year

n = 5 years

r = 8%

$\Rightarrow PVA = ??$

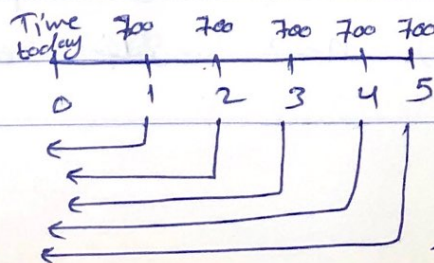
Present value of an annuity

مترجع time زيرو

مسائلنا

PV

- \$ 648.15
- \$ 600.14
- \$ 555.68
- \$ 514.52
- \$ 476.41



$$PV = \frac{FV}{(1+r)^n}$$

(single amount)

$$PVA = \text{مجموع} = \$2,794.90$$

$$PVA = \frac{CF}{r} \left(1 - \frac{1}{(1+r)^n} \right)$$

$$= \frac{700}{0.08} \left(1 - \frac{1}{(1+0.08)^5} \right) = \$2,794.90$$

نفس الجواب

* Annuity ordinary \Rightarrow تكون بنهاية فترة معينة

Annuity due \Rightarrow تكون بالبداية

تكون متأخرة

extra one period. (في حالة ال future value)

e.g : CF = \$100 \setminus \text{year (at the beginning of each year)}

n = 3 years

r = 5%

FVA due = ?? : (extra one period)

$$\textcircled{1} FV = PV (1+r)^n = 100 (1+5\%)^3 = \$115.76$$

$$\textcircled{2} FV = 100 (1+5\%)^2 = \$110.25$$

$$\textcircled{3} FV = 100 (1+5\%)^1 = ~~\$105~~ \$105$$

$$FVA_{\text{due}} = \textcircled{1} + \textcircled{2} + \textcircled{3} = \$331.01$$

(n=3 سنة) n=2 سنتين ordinary

فما سي

$$FVA_{due} = CF \left(\frac{(1+r)^n - 1}{r} \right) (1+r)$$

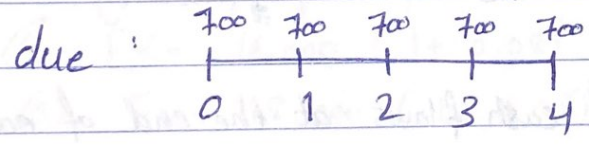
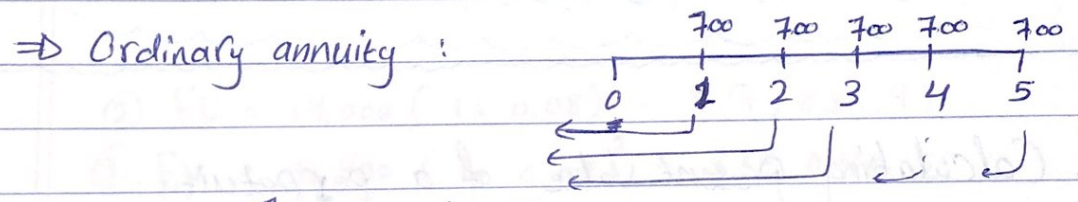
FVA Ordinary هاد زني ضفتو

$$= 100 \left(\frac{(1+5\%)^5 - 1}{5\%} \right) (1+5\%) = \$331.01$$

(نفس الجواب)

6 Calculating present value of annuity due :

eg page 226 : CF = \$700 / year (at the beginning of each year)
 r = 8%
 n = 5 years
 PVA due ?



PV is calculated due to less one period
 FV: extra 1 period

① $PV = \frac{FV}{(1+r)^n} = \frac{700}{(1+8\%)^0} = \700

② $PV = \frac{700}{(1+8\%)^1} = \648.15

③ $PV = \frac{700}{(1+8\%)^2} = \600.14



$$\textcircled{4} \quad PV = \frac{700}{(1+8\%)^3} = \$555.68$$

$$\textcircled{5} \quad PV = \frac{700}{(1+8\%)^4} = \$514.52$$

$$PVA_{due} = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} = \$3,018.49$$

بالقانون : $PVA_{due} = \frac{CF}{r} \left(1+r \right) \left(1 - \frac{1}{(1+r)^n} \right)$

ن
يعرضها
في
السؤال
(مراجعة)

$$= \frac{700}{0.08} (1+0.08) \left(1 - \frac{1}{(1+8\%)^5} \right) = \$3,018.49$$

إذا كانت $n = \infty$ بقراءة ال PV بين ما بقراءة ال FV

7] Calculating present value of a perpetuity:

* perpetuity = equal cash flows at the end of each year. $n = \infty$

$$PV = \frac{CF}{r}$$

e.g. Preferred stock

$$CF = D = \$100$$

$$r = 15\%$$

$$n = \infty$$

Stock Price today = ??

لما استنجا

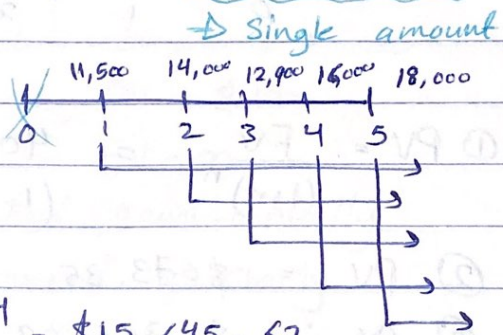
PV = ??

$$\Rightarrow PV = \frac{CF}{r} = \frac{100}{0.15} = \$666.67$$

التفسير (من النونى (العبئة 50)) :

8 Calculating future value of a mixed stream:

eg / p.229 : ~~XXXX~~



$$\begin{aligned} \textcircled{1} \quad FV &= PV (1+r)^n \\ & \quad (n=4) \\ &= 11,500 (1+8\%)^4 = \$15,645.62 \end{aligned}$$

$$\textcircled{2} \quad FV = 14,000 (1+0.08)^3 = \$17,635.97$$

$$\textcircled{3} \quad FV = 12,900 (1+0.08)^2 = \$15,048.56$$

$$\textcircled{4} \quad FV = 16,000 (1+0.08)^1 = \$17,280$$

$$\textcircled{5} \quad FV = 18,000 (1+0.08)^0 = \$18,000$$

$$\Rightarrow FV = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5} = \boxed{\$83,602.15}$$

End of the year \leftrightarrow النونى \leftrightarrow الاقام الى مظهرى لايها : كى ال

ال \leftrightarrow النونى

Beginning of the year \leftrightarrow اذا مكالنا بال \leftrightarrow مخرج رجوع (منهائس من $n=5$ لى $n=1$)

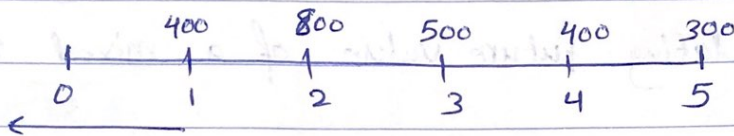
9 Calculating present value of a mixed stream:

eg: Page 230, $r = 9\%$

$n = 5$ years

End of the year

PV = ??



$$\textcircled{1} PV = \frac{FV}{(1+r)^n} = \frac{400}{(1+9\%)^1} = \$366.97$$

$$\textcircled{2} PV = \$673.35$$

$$\textcircled{3} PV = \$386.09 \Rightarrow PV = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5}$$

$$\textcircled{4} PV = \$283.37 = \$1904.76$$

$$\textcircled{5} PV = \$194.98$$

مراجعة
القوسين
Dec 10, 19

* Compounding interest more frequently than annually:

Before calculating future value or present value or present value some modifications are needed to be made:

② If interest was compounded quarterly, then ~~then~~

$$\frac{r}{4}, n \times 4 \Rightarrow (4n)$$

① If interest was compounded semi-annually, then

$$\frac{r}{2}, n \times 2 \Rightarrow (2n)$$

③ If interest was compounded monthly, then:

$$\frac{r}{12}, n \times 12 \Rightarrow (12n)$$

④ If interest was compounded weekly, then:

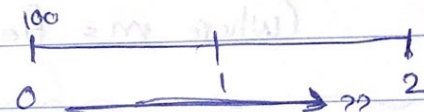
$$\frac{r}{52}, n \times 52 \Rightarrow (52n)$$

* Note: Started = r .

eg page 233: $n = 2$ year . $PV = \$100$
 $r = 8\%$. $FV = ??$

a- If interest was compounded semi-annually

b- If interest was compounded quarterly.

[a]: $\frac{r}{2} = \frac{8\%}{2} = 4\%$ 
 $m = 2$ (circled in pink) = عدد الفترات

$$n = 2 \times 2 = 4 \text{ periods.}$$

$$\Rightarrow FV = 100 (1 + 4\%)^4 = \$116.99$$

[b]: $\frac{r}{4} = \frac{8\%}{4} = 2\%$

$$n = 4 \times 2 = 8 \text{ periods}$$

$$\Rightarrow FV = 100 (1 + 2\%)^8 = \$117.7$$

Frequency الـ \Rightarrow كلما زاد الـ frequency \Rightarrow زاد الـ FV
 $(117.7 > 116.99)$
 \downarrow quarterly \downarrow semi-annually
لا تتراكم الفوائد بزيادة

* Nominal interest rate versus effective annual rate:

⊛ Nominal interest rate = stated interest rate
= annual interest rate

$r = 8\%$ = annual interest rate
= stated interest rate

⊛ Effective annual rate = interest rate
↓
actually paid

$$\Rightarrow \text{EAR} = \left(1 + \frac{r}{m}\right)^m - 1$$

(where m = frequency).

e.g. page 238: EAR = ??

$r = 8\%$ = nominal interest rate.

a - when interest is compounded annually?

b - " " " " semi-annually?

c - " " " " quarterly?

$$\text{a) } \text{EAR} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{8\%}{1}\right)^1 - 1 = \boxed{8\%}$$

$\therefore \Rightarrow$ If interest was compounded annually ($m=1$), then
EAR = nominal interest rate.

- (colo 5, \Rightarrow 8%) \rightarrow

$$b) \text{ EAR} = \left(1 + \frac{8\%}{2}\right)^2 - 1 = \boxed{8.16\%}$$

$$c) \text{ EAR} = \left(1 + \frac{8\%}{4}\right)^4 - 1 = \boxed{8.24\%}$$

* Special applications of time value of money:

* Loan ~~amortization~~ amortization schedule.

eg Page 241

Loan amount = \$6,000

$r = 10\%$

$n = 4$ years

PMT = ??

in equal payment

→ Payment cash flow.

(مفعول متساوي) : annuity

$$PVA = \frac{CF}{r} \left(1 - \frac{1}{(1+r)^n}\right)$$

$$\$6,000 = \frac{CF}{0.1} \left(1 - \frac{1}{(1+0.1)^4}\right) \Rightarrow CF = \$1892.82$$

Loan PMT = interest + principal

PMT

المبلغ الذي ستدته = Loan PMT - interest

Year	Beg. Bal	loan Payment	interest (10%)	Principal	End. Balance
1	\$6,000	\$1,892.82	\$600	\$1,292.82	\$4,707.18
2	\$4,707.18	\$1,892.82	\$470.72	\$1,422.1	\$3,285.08
3	\$3,285.08	\$1,892.82	\$328.51	\$1,564.31	\$1,720.77
4	\$1,720.77	\$1,892.82	\$172.08	\$1,720.74	- 0 -

مفعول متساوي

لا يتبقى شيء ولا يتبقى شيء

ملا > قايح د قواسم و ~~...~~ : الما لاسايت

⊗ Interest = Beg. Balance × interest rate.

⊗ principal = (Loan) - interest
PMT

⊗ End. Balance = Beg. Balance - principal.

⊗ ال End. Bal. سيق ال ال Beg. Bal. في السنة التالية.

⊗ ال زيادة في ال Assets تعبير cash outflow

⊗ تعبير P4-18

b) expenses = fixed + variable
exp. are not all variable

d) \$180,000 \$230,625 \$285,500
overstated understated of
of Profit profit.

بالقارة
الطريق
الاسوي

\$167,325 \$295,480 \$383,640